Quantifying confidence in probability assessments

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A modest proposal

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- Risk assessments often close with the question:

How confident are you in your assessment?

Not confident at all [1] [2] [3] [4] [5] Very confident

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I think it would be inadvisable to base an action on the answer to this question.

- My modest proposal is to replaced this by:

How confident are you in your assessment?

Unlikely to change much over the next five years 1 2 3 4 5 Likely to change a lot over the next five years

A modest proposal (cont)

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2. It aligns more closely with the needs of policymakers, for whom the pressing question is often "Do we act now, or do we delay for another cycle?"

3. It can be quantified using the experts' assessment of the relevance of the historical record. Notably the length of the relevant record compared to the prospective period.

‘Likelihood’ (UK NRA definition)

The ‘likelihood’ $\mathcal{L}$ of a hazard class is the probability of at least one major event happening in the next five years. Or, if N is the number of major events, $\mathcal{L} = \Pr\{N(0, 5] > 0\}$.

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3. So currently, the future likelihood is a random quantity $\mathcal{L}(h, F; k)$, and we can compute its distribution function

$$F_{\mathcal{L}}(\ell \mid h; k) := \Pr\{\mathcal{L}(h, F; k) \leq \ell \mid h\}.$$

‘Likelihood’ (UK NRA definition) (cont)

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The proposed definition is generic, but the calculation becomes nearly trivial under the model that large events follow a Poisson process. In this case, in the simplest treatment, the experts need only decide how far back to go while not violating homogeneity.

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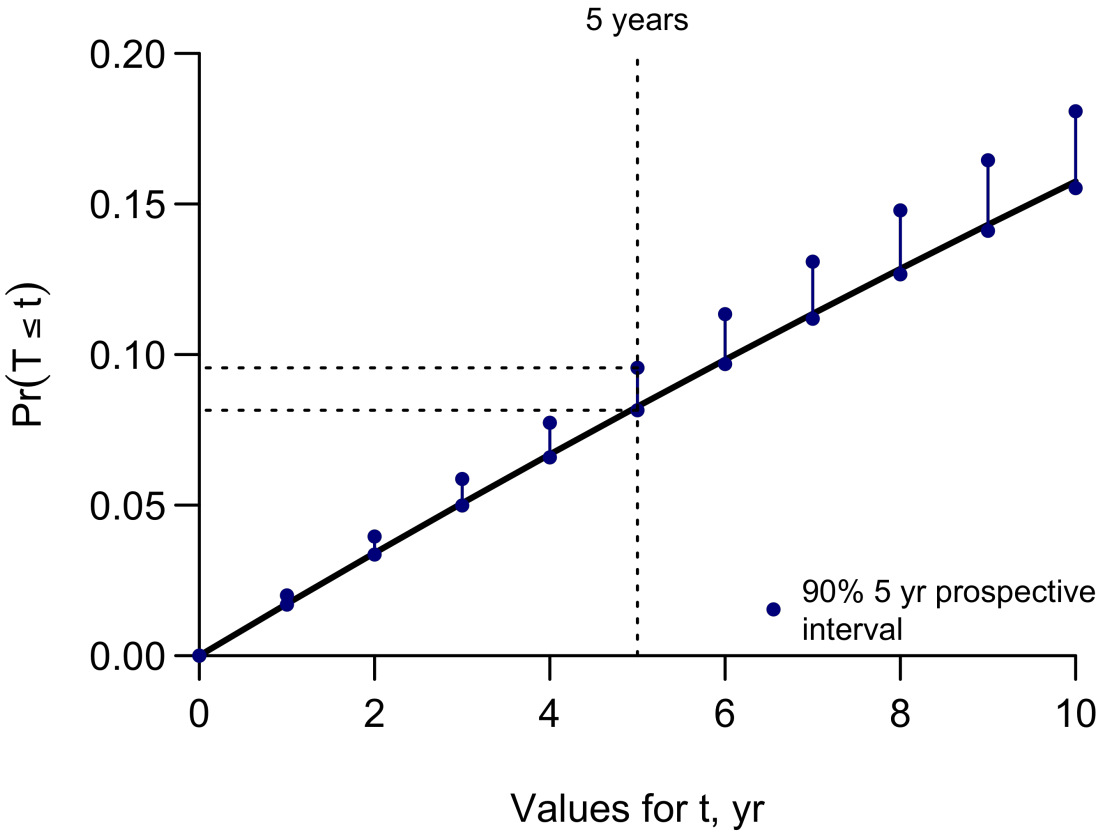

That looks like a firm-ish 0.083 (90% 5-year PI: 0.081, 0.096).

Illustration: UK oil refinery/facility fires

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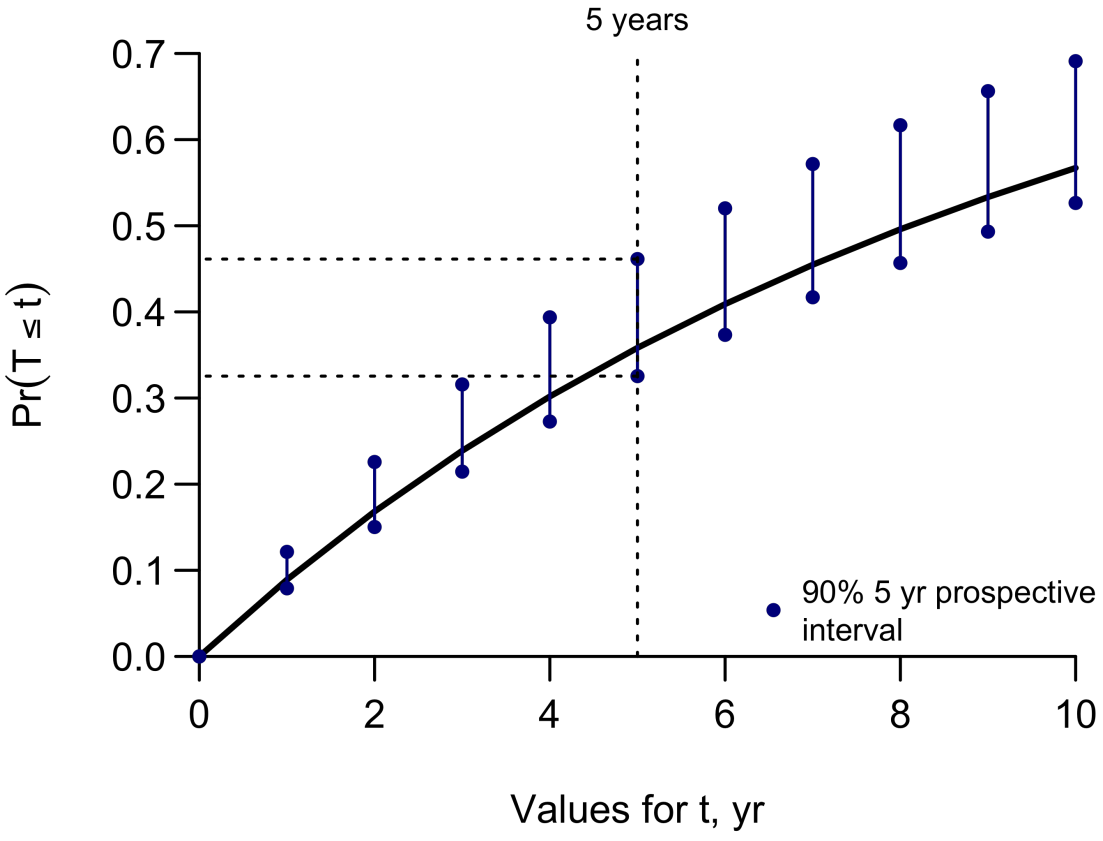

Soft-ish 0.36 (90% 5-year PI: 0.33, 0.46). Illustration only!!

Appraisal

1. Operationalizing 'confidence' is a good idea.

2. Using the quantiles of $F_{\mathcal{L}}$ seems to be a natural approach.

3. Stationary process modelling reduces the assessment process to two values: the length of the relevant historical record, and the number of events during that time.

4. This can adapt to non-stationary processes simply by reducing the length of the relevant past, but of course the intervals are wide.